An Introduction to Shot Noise in Laser Beams

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Abstract

Shot noise limits the sensitivity of the LIGO interferometers at frequencies above about 100Hz. I show how Poisson statistics combined with Heisenberg's uncertainty ${LALHOME} principle can be used to deduce the shot noise sensitivity limit in the LIGO detectors.

Poisson Statistics

- Govern systems where there is some event whose probability of occuring in a fixed time interval of length t is constant.

Figure 1: The cars in the left (westbound) lane are approximately governed by Poisson statistics. *Why only approximately ?*

- I measured 17 cars in 1 minute

The Poisson Distribution

$$p(n) = \frac{\mu^{-n} e^{-\mu}}{n!} \qquad (1)$$

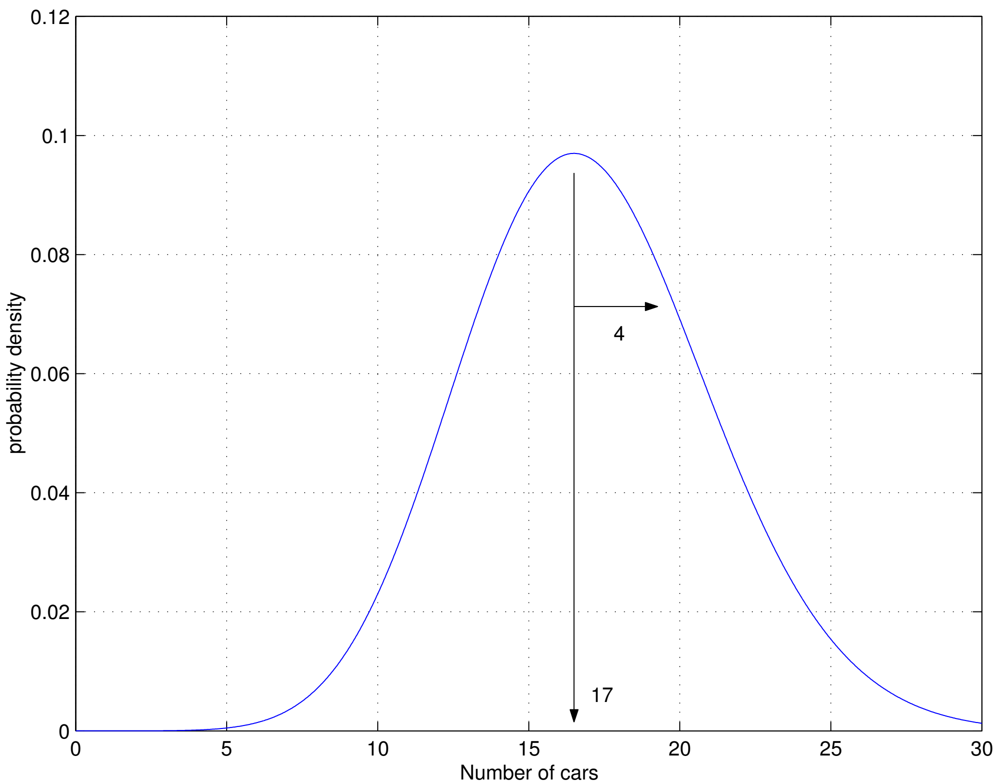

Figure 2: Poisson distribution with a mean of 17

$$\sigma_n = \sqrt{\mu} \qquad (2)$$

Laser beam model 1: Particles

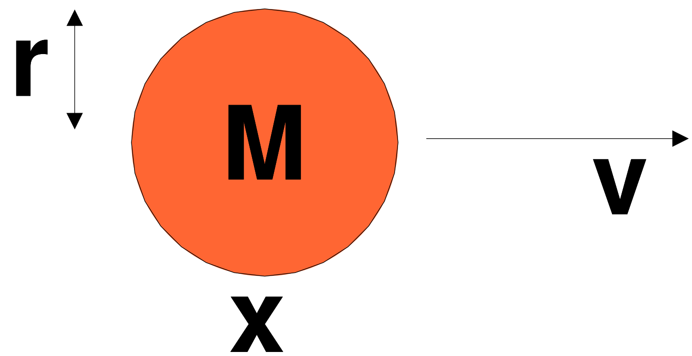

Figure 3: We might think of a laser beam as made up of particles. *Problems ?* Particle properties:

- A finite size
- Sharp edges
- A position
- A velocity
- Can travel through a vacuum
- A mass

Laser beam model 2: Waves

Figure 4: We might think of a laser beam as made up of waves. *Problems ?*

Wave properties

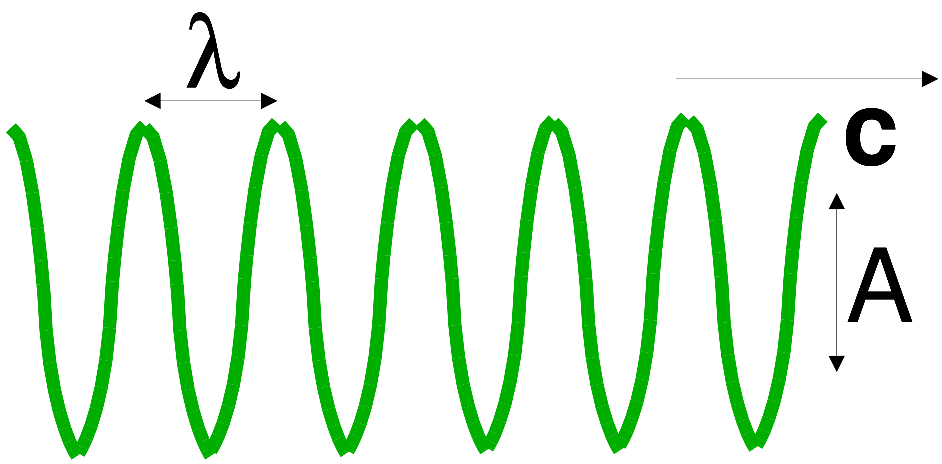

Figure 5: An idealized wave. Wave properties:

- A wavelength λ
- An amplitude
- Infinite extent
- Travel in a medium
- A velocity

Laser beam model 3: Wave packets

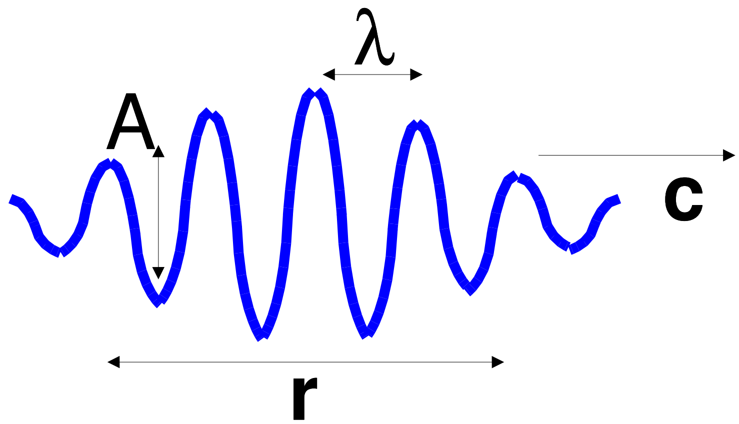

Figure 6: A wave packet. Properties:

- A finite size with fuzzy edges
- A phase ϕ
- A velocity c
- A wavelength *range* $\lambda \rightarrow \lambda + \Delta\lambda$
- An energy E

Laser beams of wave packets

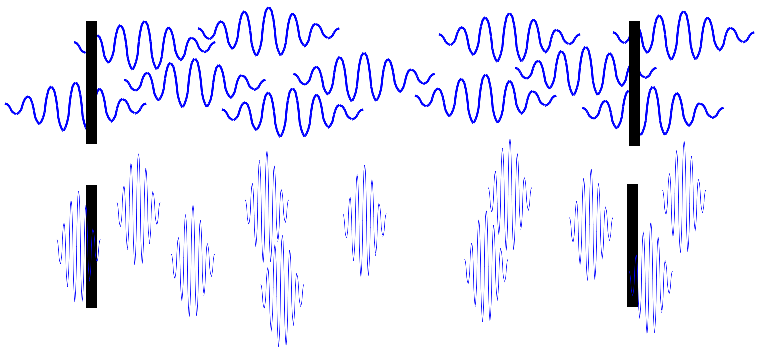

Figure 7: *top:* more wave-like. *bottom:* more particle-like. Between the black markers:

- Number of particle is $N \pm \sigma_N$.
- Phase of wave is $\phi \pm \sigma_\phi$.

Heisenberg's uncertainty principle says

$$\sigma_N \sigma_\phi = 1. \tag{3}$$

How Many Wave Packets ?

Planck's law relates energy E_γ of a wave packet to its wavelength λ.

$$E_\gamma = \frac{hc}{\lambda}, \quad (4)$$

where $\mathrm{c} = 3.0 \times 10^8 \mathrm{ms}^{-1}$ is the speed of light and $\mathrm{h} = 6.6 \times 10^{-34} \mathrm{Js}$ is Planck's constant.

A laser beam of power $\mathrm{P}_\gamma = 5\mathrm{W}$ with wavelength $\lambda = 1\mu\mathrm{m}$ carries $5\mathrm{J}$ of energy past some point per second. The number of photons per second N_γ is

$$\mathrm{N}_\gamma = \frac{\mathrm{P}_\gamma}{\mathrm{E}_\gamma} = \frac{5 \times 10^{-6}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 3 \times 10^{19}. \quad (5)$$

Bring on the Poisson Statistics

Suppose photons in a laser beam obeyed poisson statistics!

$$\sigma_N = \sqrt{N_\gamma} = \sqrt{3 \times 10^{19}} = 5 \times 10^9 \tag{6}$$

So we only know the *photon flux* to within ± 5 billion! What about the phase? From the uncertainty principle of equation 3,

$$\sigma_\phi = \frac{1}{\sigma_N} = 2 \times 10^{-10} \text{radians} \tag{7}$$

So the phase of the laser waveform is uncertain to ± 0.2 nanoradians.

The uncertainty in the phase of a laser beam due to quantization of light into wave packets is called *shot noise*. It is a small effect, but *hard to get around*.

Shot Noise in LIGO Interferometers 1.

LIGO interferometers measure the *relative phase* between two light beams recombining at a beamsplitter.

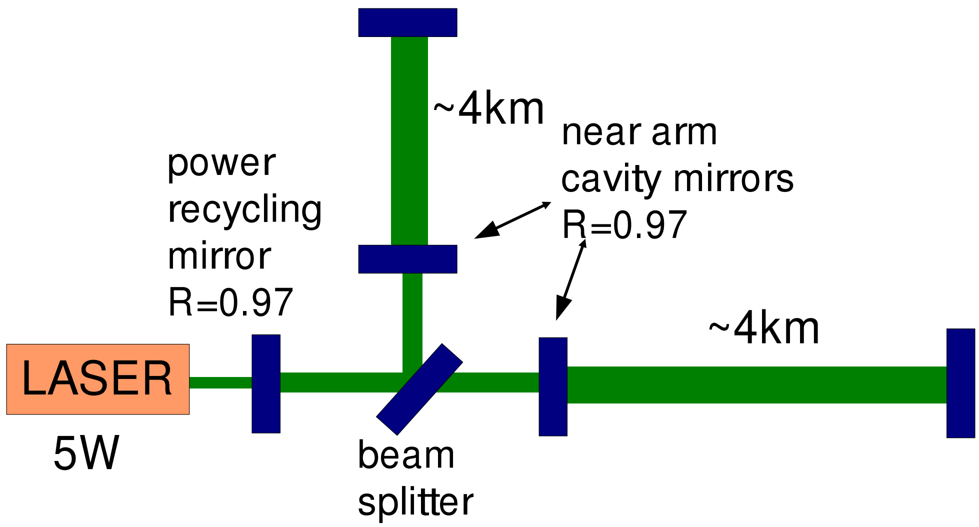

Figure 8: A simplified schematic of a LIGO interferometer

Notice that the laser power at the beamsplitter will be *greater* than the 5W emmitted by the laser. This is because each photon emmitted by the laser bounces back and fourth between the recycling mirror and the arms multiple times, about 60 on average, before exiting the interferometer.

Shot Noise in LIGO Interferometers 2.

If a 5W light beam bounces 60 times in a cavity before exiting, the total number of photons in the cavity multiplies by 60. Therefore the number of photons at the beam splitter is

$$\mathrm{N}_{\gamma}^{\mathrm{LIGO}} = 60 \times 5 \times 10^{-6}/\mathrm{hc} = 2 \times 10^{21}, \qquad (8)$$

where I used the LIGO laser wavelength of $1\mu\mathrm{m}$.

Carrying on as before, the phase noise at the beamsplitter $\sigma_{\phi}^{\mathrm{LIGO}}$ is

$$\sigma_{\phi}^{\mathrm{LIGO}} = \frac{1}{\sqrt{\mathrm{N}_{\gamma}^{\mathrm{LIGO}}}} = 2 \times 10^{-11}\mathrm{radians} \qquad (9)$$

From Shot Noise to Strain Sensitivity 1.

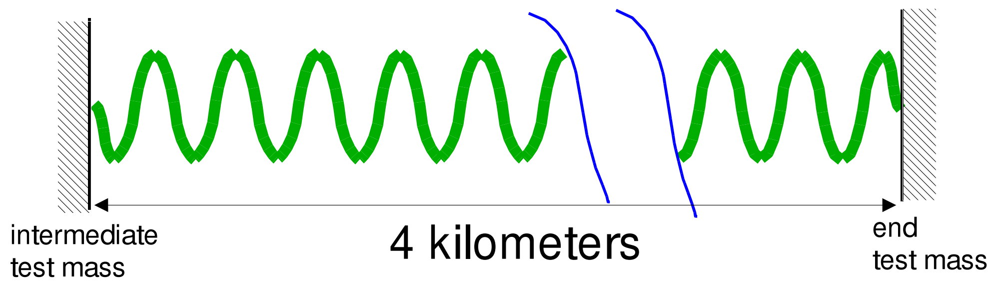

Figure 9: Light resonating in a LIGO arm

You may not (yet) appreciate how amazingly good a shot noise of $\sigma_\phi = 2 \times 10^{-11}\mathrm{radians}$ is. Consider: 2π radians is a full cycle of the wave. For LIGO this is $1\mu\mathrm{m}$.

Arm strain is defined as the change in the length δL of (in this case) an interferometer arm divided by the original length L. How much change in the length of the arm would produce a phase shift of $2 \times 10^{-11}\mathrm{rad}$? If 2π phase is a micrometer, then δL corresponding to the LIGO design phase noise is

$$\delta \mathrm{L} = 10^{-6}\mathrm{m} \times \frac{2 \times 10^{-11}}{2\pi} = 3 \times 10^{-18}\mathrm{m} \qquad (10)$$

From Shot Noise to Strain Sensitivity 2.

What is L? L is NOT the arm length of the interferometer. As was the case with the beamsplitter, light bounces back and forth between the intermediate and end test masses many times before exiting back towards the beamsplitter. The average number of bouces per arm is about 50. This boosts the effective arm length by a factor of 50!.

The interferometer strain δh equivalent to a phase noise of $2 \times 10^{-11}\mathrm{m}$ is

$$\delta\mathrm{h} = \frac{\delta\mathrm{L}}{4\mathrm{km} \times 50} = 1.5 \times 10^{-23} \qquad (11)$$

If this is really the loudest noise source in the interferometer then we are sensitive to gravity waves that produce a strain of 10^{-21} with a signal to noise ratio of about $10^{-21}/10^{-23} = 100$.

If....

Before boasting to your friends that the instrument you worked on last summer could measure distance changes of $1/1000$ of a proton diameter, remember the following:

- There are $many$ other much louder noise sources
  - laser frequency noise
  - electronics noise
  - noise from very low frequencies mixed up to few $\times 100\text{Hz}$ .....

The LIGO design is such that these noise sources should be smaller than 1/10 of the shot noise level each at the frequencies LIGO is designed to detect (a few hundred Hz). But making this a reality will mean *lots more hard work*. Finally:

- The full LIGO detectors currently only work for minutes at a time

Conclusions

- Laser beams can be understood as streams of wave packets whose peaks and troughs are lined up
- Shot noise arises from fluctuations in the number of particles passing a point in the beam with time
- The number of photons passing a point in a fixed time interval obeys poisson statistics
- Higher frequency photons are sharper, more particle–like wave packets
- Lower frequency photons are less sharp, and more wave–like
- Heisenberg says, you can't measure both the number of photons passing a point and the phase of the wave at that point at the same time
- The above restriction sets the fundamental strain noise of the LIGO detectors to $\delta \mathrm{h} \sim 2 \times 10^{-23}$.